MANUAL

OF

STEREOSCOPY

For Use With Abrams

CB-1 Stereoscope

CF-8 Stereoscope

HF-2 Height Finder

Photogrammetric Computer

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SECTION I
Introduction

1-1 INTRODUCTION. This manual was written to provide a basic understanding of Stereoscopy and at the same time provide proper operating instructions for the Abrams stereoscopes, height finder, and photogrammetric computer. These instructions and examples will enable an inexperienced person to accomplish three-dimensional viewing and make basic elevation measurements.

SECTION II
Theory

2-1. STEREOSCOPY. Stereoscopy is the science and art of producing three-dimensional effects and the methods employed to produce these effects.

In order to see an object stereoscopically, it must be viewed from two different positions parallel to it, thus rendering a three-dimensional view. Any object that occupies space has three dimensions—length, width and depth. This is illustrated in Figure 1, which shows how one eye registers the length and width of an object, while the other eye registers the length, width and depth. It is the parallax created between B & B' that produces the effect of third dimension. Parallax is the apparent displacement of the position of an object with respect to a reference point which is caused by a shift in the point of observation.

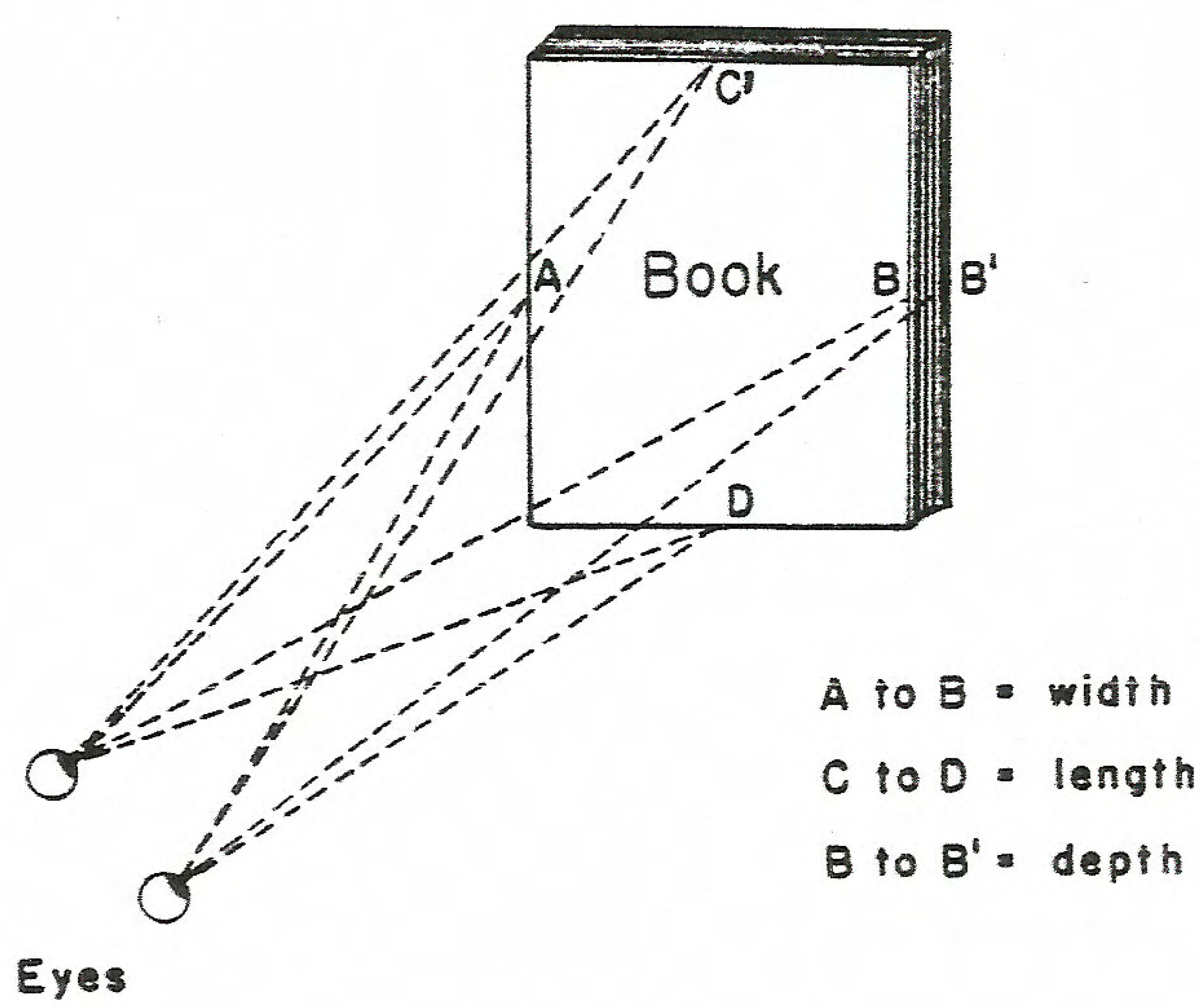


Figure 1—Seeing Three Dimensions

2-2. APPLICATIONS. The science of stereoscopy, from a military standpoint, first played an important part during World War II in that ground information of distant or enemy territory was obtained by some means other than personal reconnaissance. The very scope of the conflict and the speed with which it moved, due to its mechanized nature, required the obtaining of intelligence information at a rate never before attained in military history. This information was obtained through the use of overlapping (stereoscopic) pairs of vertical aerial photographs. On every adequate photographic day, strips of aerial photographs were taken over selected portions of the enemy territory and thoroughly scrutinized by the aid of the stereoscope for detection of enemy movements, camouflage, bomb damage, et cetera. Thus was born the art of PI (Photo Interpretation).

The art of stereoscopy is also playing an important part in modern map-making. Not only does it have a great military value, but many private agencies are using aerial photographs and the stereoscope for solving some of their most technical problems, such as: city planning, methods of soil conservation, location of geological formations, better transportation, facilities, et cetera. The stereoscope is here to stay, and will continue to be an important piece of equipment for photo interpreters, aerial photographers, engineers, photogrammetrists, field geologists and forest engineers.

2-3. AERIAL PHOTOGRAPHS. An element of stereo-vision is brought about when an airplane flies over an area, taking photographs with the axis of the camera lens vertical. The exposures are taken so that an area on the photograph also appears on the photograph next to it. The overlapping area is suitable for stereoscopic study, only when the photographs have sufficient stereoscopic coverage and are properly set up to show the relief, or relative elevation and structures that appear in the area. Aerial photographs are usually taken so that they have a minimum overlap of 50%. This result is known as a spatial model. Unless every photograph has this percentage or more of overlap, it is impossible to have complete stereoscopic coverage in a line of flight. A sure method of obtaining stereoscopic coverage is to run an overlap of about 60% in line of flight so that no two overlapping photographs ever have less than 50%. In order to have usable stereoscopic coverage of an area, there must be a minimum overlap of 15% between flight lines. Less than this could result in distortion of the image or an insufficient coverage in cases of tip, tilt or crab.

Care should be taken in positioning photographs for viewing especially in the cases of rough terrain or photographs containing deep shadows, in which case, when improperly viewed, the relief may appear in reverse; that is, the valleys will appear to be ridges and mountain peaks appear to be pits. This illusion is called pseudoscopic effect. The occurrence of this illusion can be greatly minimized if the photographs are always viewed with the shadows towards the viewer.

2-4. STEREOSCOPIC VISION.

Stereoscopic vision (usually referred to as stereo-vision) is necessary in order to obtain stereoscopic effects. Stereo-vision is the ability of a person to combine two perspective images of an object, in such a manner as to create a mental impression of relief or three-dimensional effect. Each of the two perspective images of the same object must be produced from a somewhat different angle, such as two photographs taken from different camera stations. The resultant photographs are known as a stereo-pair.

It thus follows that persons with monocular vision cannot see stereoscopically. This is not to say that all persons having binocular vision can see stereoscopically either. An elderly person often has much more difficulty in seeing stereoscopically than a younger person, because the eye muscles have become set and will not relax easily. Some people can never see stereoscopically from a stereo-pair even though they have normal vision in both eyes.

2-5. STEREOSCOPES.

The simplest optical instruments for viewing objects in three dimensions are stereoscopes. These may be of the mirror *(reflecting)* type, the prism type or the lens *(refracting)* type. The first recorded stereoscope was a mirror type developed by Robert Wheatstone in 1838. A few years later, Sir David Brewster developed a lens stereoscope.

The stereoscope is an optical apparatus which enables us to look at the same time upon two photographic pictures nearly the same, but taken from a small difference of angular view. Each eye looks upon one picture only; and as in ordinary vision, two images are conveyed to the brain which unite into one, the objects being represented under a high degree of relief. The stereoscope is constructed in accordance with the visual phenomena which convey to the mind impressions of the relative forms and positions of an object. When a near object having three dimensions is looked at, a different perspective representation of it is seen by each eye; in other words, there is distinct binocular parallax. (See Figure 1). Certain parts are seen by the right eye (the left being closed), that are invisible to the left eye, and vice versa. These two visual impressions are simultaneously perceived by both eyes, and are combined into one image, producing the impression of perspective and relief.

If then, true right-and-left monocular pictures of an object are presented to the two eyes so that the optic axes are directed to the eyes at the same angle of convergence that was directed to the object by the camera, a solid or three-dimensional image of the object will be seen.

In monocular vision, all that can be determined about the position of an object is its relative direction in the field of view. Binocular vision, however, gives some estimate of the distance and depth perception provided that the object is not too far away as compared with the interpupillary distance of the observer. Interpupillary distances range from 1.97 inches to 2.85 inches, however, the average interpupillary distance is approximately 2.625 inches. It has been found that it is impossible to distinguish between objects if the difference of the angle of convergence is less than about 20 seconds of arc, although in some extraordinary persons, this minimum angle of depth perception can be as low as 10 seconds of arc.

SECTION III

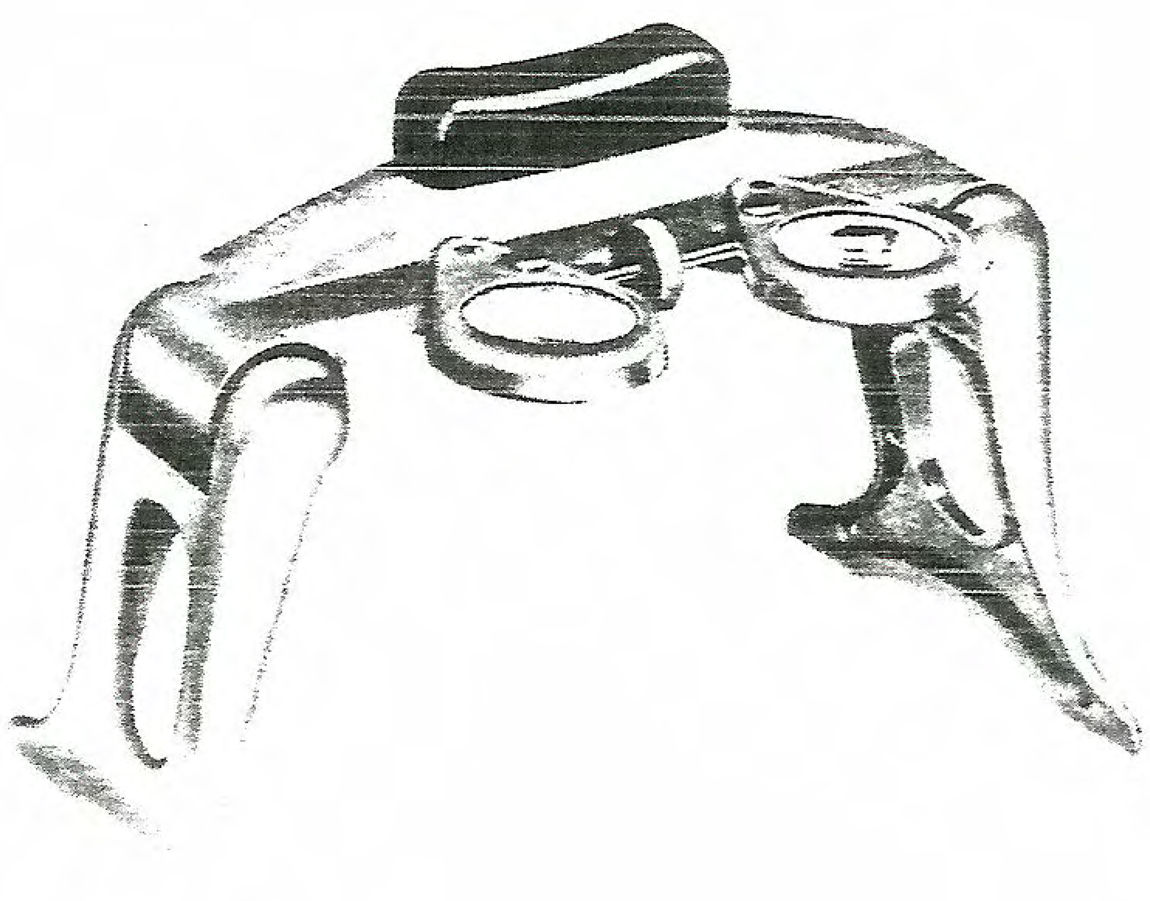
Photo Interpretation Instruments

3-1. GENERAL. Abrams Instrument Corporation's equipment has been designed to be as compact as possible, but yet provide maximum versatility. The stereoscopes and height finder are supplied with carrying cases which facilitates field use. The height finder has been adapted to both stereoscopes which improves its utility. Although rugged in design, this equipment requires proper care and adjustment.

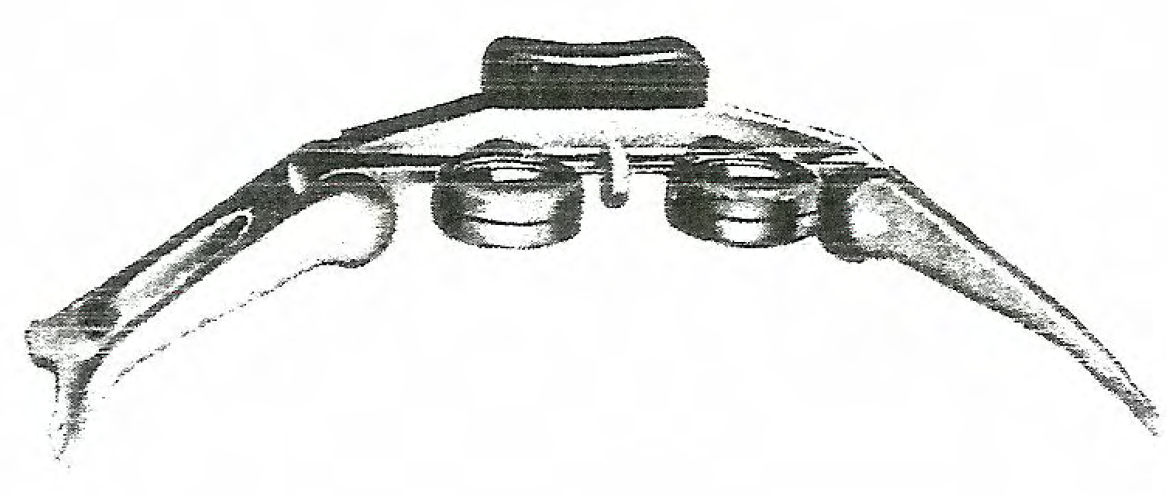
3-2. CB-1 STEREOSCOPE. The Abrams Model CB-1 Stereoscope is a precision instrument designed for use with stereoscopic pairs of vertical aerial photographs and is capable of magnifications of either two or four power.

As the instrument is taken from the carrying case, the two pairs of lenses are together and the legs are folded up under the body of the instrument. Swing each leg out until it snaps into the first stop at approximately a right angle to the body. Then push each of the lenses in the lower pair downward and backward until it snaps up into the out-of-way position. The instrument is then ready for use with two power magnification. (See Figure 2)

To use the four power magnification the legs must be set in the most extended position. Grasp the legs with each hand, and press with the thumbs on the stereoscope body, until the leg locks disengage. Then rotate each leg outward as far as it will go. By releasing the thumb pressure, the legs will lock in the proper location. The lower pair of lenses are then moved downward and forward until the two sets of lenses are together. The instrument is now ready for four power observation. (See Figure 2)



2 Power Position



4 Power Position

Figure 2 – CB-1 Stereoscope

Each set of lenses, 4¾ inches focal length, 2 power magnification, meniscus type, are easily adjusted for individual interpupillary requirements by rotating the knurled wheel in either direction as required. A scale calibrated from 50 to 75 mm on the right lens allows for definite settings.

The anti-fatigue head rest allows the operator to rest his head with comfort as he scans stereoscopic prints.

The CB-1 Stereoscope in the 2 power position is adapted for use with Abrams Model HF-2 Height Finder. With this combination of instruments, it is possible to determine heights of objects or variation of heights of terrain features in overlapping vertical aerial photographs.

When the instrument is to be placed in the case, put the lenses together in the four power position. Then, grasping the stereoscope legs with each hand, place thumbs on body and press back until the leg locks are disengaged. Rotate the legs under the body as far as they will go. Place the instrument in the case with the legs on the bottom and the lenses to the back of the case. This permits the head rest to fit into the well in the cover and prevents the instrument from shifting in the case.

The CB-1 Stereoscope body and legs are constructed of durable nylon moldings which, with normal usage, will give trouble-free service for many years. The head rest is of long-wearing rubber. If soiled by oil or perspiration, the instrument is easily cleaned by using soap and water.

3-3. CF-8 STEREOSCOPE. The Abrams Model CF-8 Stereoscope has been designed for use with vertical aerial photographs. It is a great convenience for aerial photographers, photo interpreters, engineers and photogrammetrists who desire a small compact instrument which will fit easily, with its case, into the ordinary vest pocket.

This stereoscope is furnished complete with leather case. It has interpupillary adjustment permitting from 55 to 75 mm. This adjustment requires only a slight pull to separate the lenses to the desired working distance. There are no screws or nuts to come loose. The lenses are 4¾ inches focal length, 2 power magnification, meniscus type, producing clear, sharp images without distortion. Steel wire legs support the stereoscope forming a loop without points that will protect the photographs from scratching. They fold easily along the sides of the lens holders to protect the instrument while being carried.

Remove the instrument from the case and swing the legs into a locked position. Separate the lenses and adjust for interpupillary distance by holding up to eyes or by adjusting to a determined length on the scale. It is quite important to get this distance correct for it avoids eye strain. (See Figure 3)

When folding the instrument, close the lens holder together, place two thumbs on the front corner of the lens holder and disengage the legs by applying pressure with the fingers. After the legs have been disengaged from the frame, turn the left leg up and over the lens holder and the right leg down and under the lens holder. Make sure the lens holders are together so that the legs will form a locking device. Place into the case.

The CF-8 Stereoscope can be used with the Abrams Model HF-2 Height Finder. Instructions for use are identical for both stereoscopes.

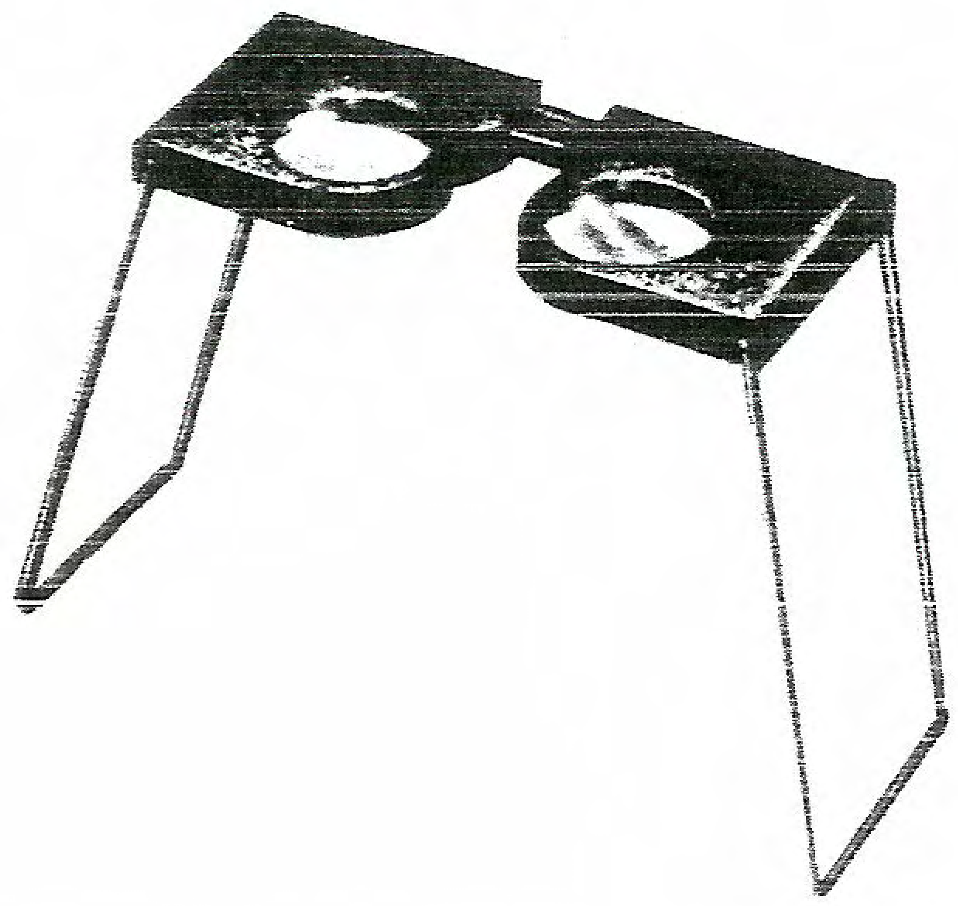


Figure 3 – CF-8 Stereoscope

3-4. HF-2 HEIGHT FINDER. The Model HF-2 Height Finder is a measuring device designed for use with Abrams Stereoscopes, Models CB-1 and CF-8, for the purpose of delineating topographic detail and determining elevations from a pair of vertical aerial photographs.

When two overlapping aerial photographs are properly oriented and viewed stereoscopically, the image displacement (parallax) can be properly measured and by means of simple computations transformed into elevation values.

The HF-2 has been devised and constructed to accurately measure parallax differences to the accuracy of .01 mm. by employing the 2 power Abrams Magnifying Stereoscopes CB-1 and CF-8.

The HF-2 is composed of the following major parts: frame left-hand lens (reference dot), right-hand lens (moving dot), knob (calibrated in millimeters), studs, and spring clips. (See Figure 4)

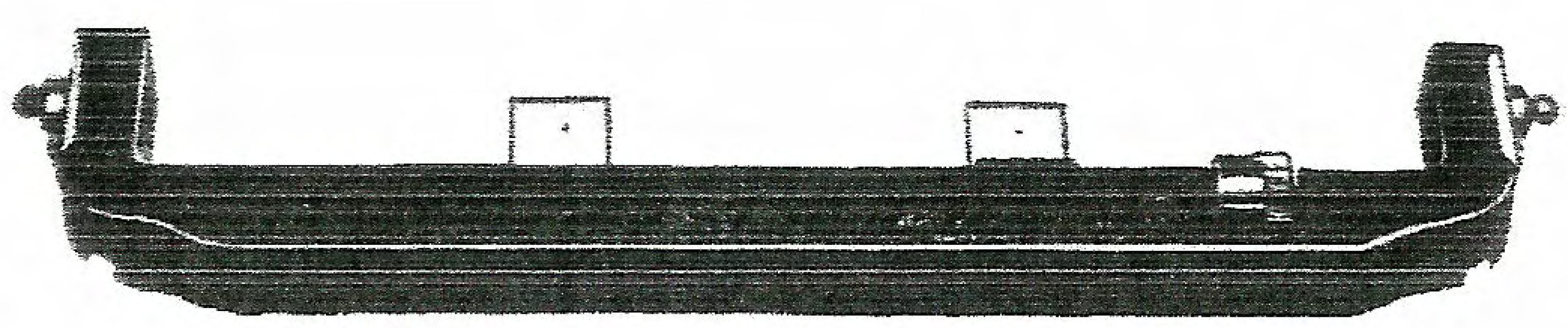
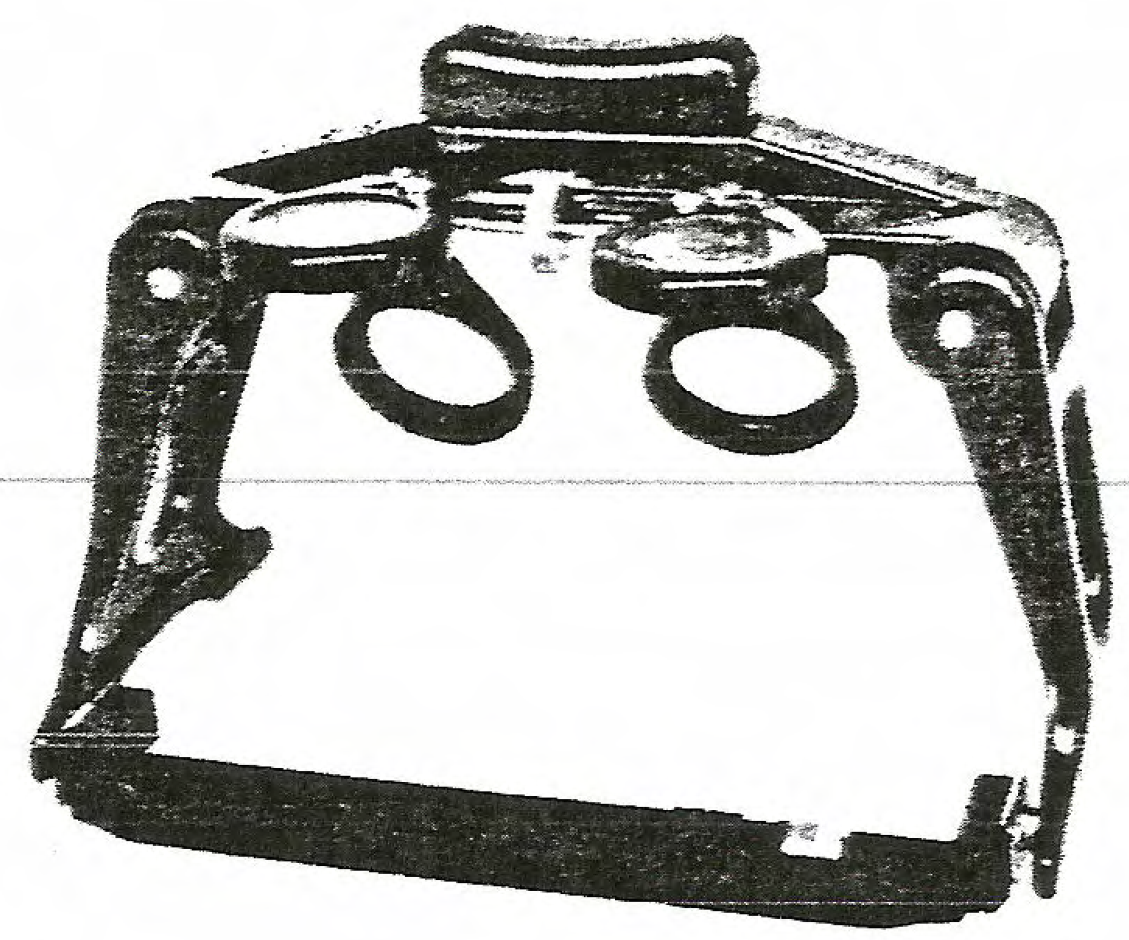


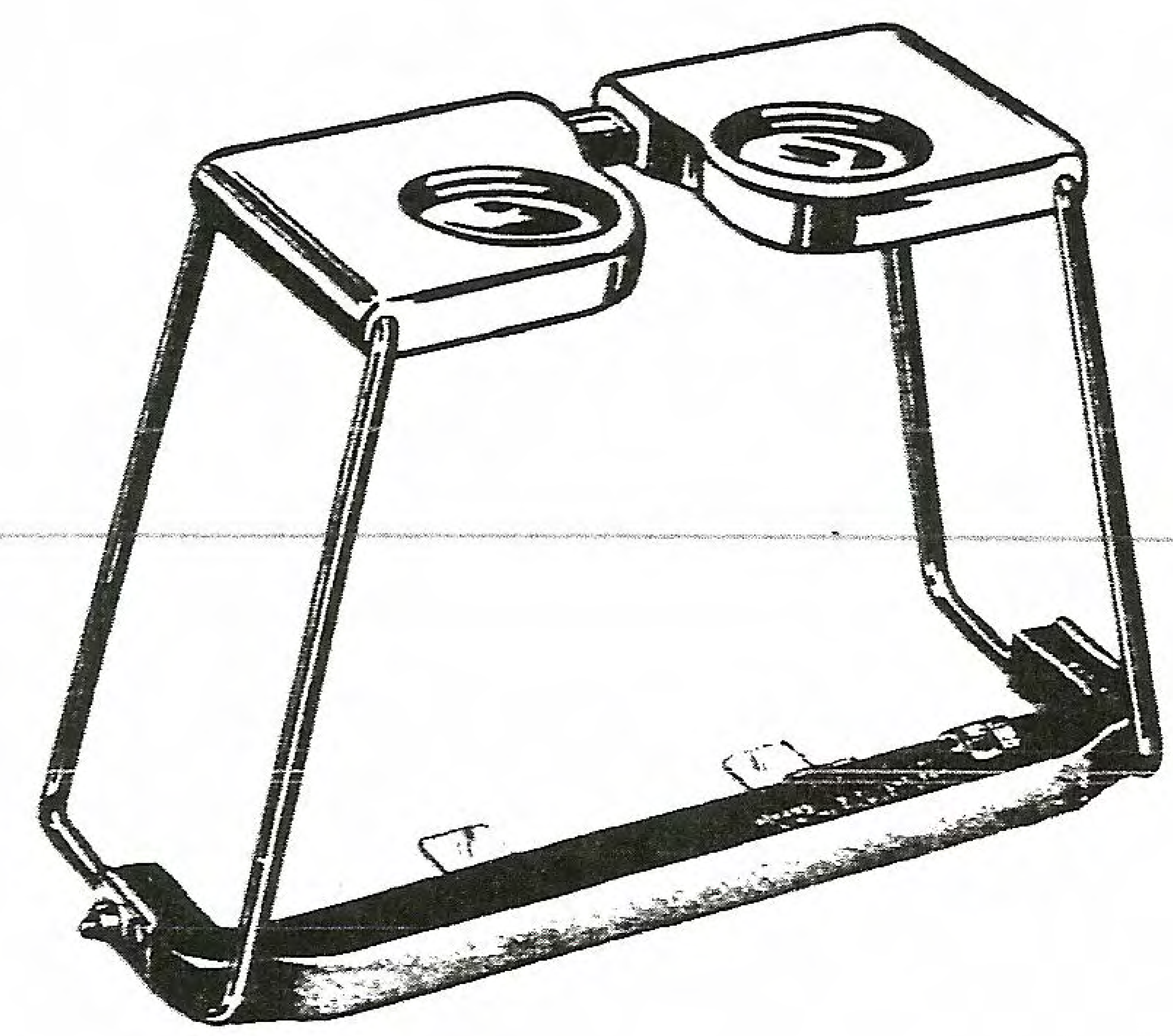
Figure 4 – HF-2 Height Finder

The reference dot is engraved on the underside of the clear plastic lens. Loosening the two screws on the underside of the instrument, which hold the reference dot lens, will permit the lens to be adjusted to eliminate any "y" parallax which may result from attaching the HF-2 to the stereoscope. The moving dot is also engraved on the underside of the clear plastic lens and is actuated by the knob. The metric scale directly measures the parallax displacement which is determined by turning the knob. It is capable of a total reading of 20 mm.

To assemble the HF-2 to the Model CB-1 Stereoscope, position the stereoscope in the 2 power position on a flat surface and align the studs of the HF-2 with the holes located on the inside of the stereoscope legs. Using the thumb and forefinger, squeeze the stereoscope leg against the HF-2 frame until the stud snaps into the hole. Repeat for the other leg. (See Figure 5)



CB-1



CF-8

Figure 5 – Stereoscopes with Height Finder

To assemble the HF-2 to the Model CF-8 Stereoscope, place the HF-2 on the cross member of the stereoscope legs with the knob to the right. Push down the ends of the HF-2 to engage the spring clips. Slide the HF-2 as far toward you as the construction of the stereoscope legs will permit. This will place the dots directly below the stereoscope lenses. (See Figure 5)

Note for the Color Blind: It has been found the color blind people cannot distinguish the red dots on the clear plastic lenses of the HF-2, as they see red as a shade of grey which blends into the greys of the photographs. However, since most of these people can distinguish yellow much better, the red pigment can be removed from the engraved dots by using a fine needle and the dots refilled with yellow grease pencil.

3-5. PHOTOGRAMMETRIC COMPUTER. The Abrams Photogrammetric Computer has been designed to solve parallax equations required in aerial photographic surveying and mapping. The computer is made of durable plastic. (See Figure 6)

The A scale is given in millimeters, and the B scale is given in inches. These two scales are interchangeably used to represent the focal length of the camera lens and the distance between photo centers. The two scales together provide the conversion table for converting millimeters to inches and inches to millimeters. The B scale is also used to convert parallax measurements to vertical elevations.

The C scale is given in feet and represents the scale of the photo in feet per inch and altitude of the aircraft above the ground in feet.

The D scale represents feet per .001 of an inch.

The E scale represents feet per millimeter.

The F scale represents feet per .01 of a millimeter.

The D, E and F scales are used to convert parallax measurements into differences of elevations, the answer being in feet.

The B, C and D scales are also used for multiplication and division.

The index is shown at the beginning of the B scale.

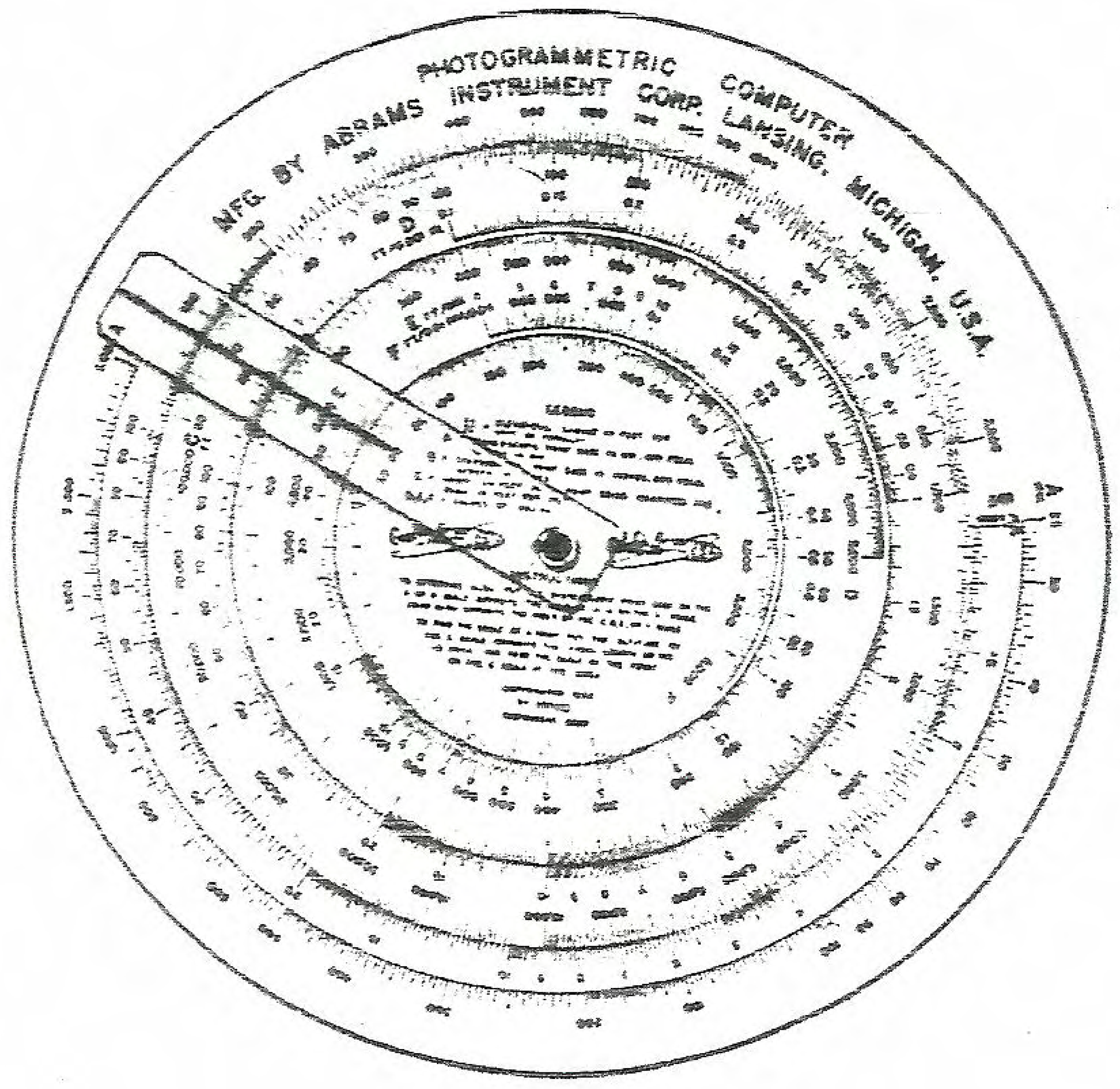


Figure 6—Photogrammetric Computer

To determine photo scale (S), divide the altitude of the aircraft above ground ($H - h$) by the focal length of the lens (f).

> Put the altitude of the aircraft above the ground ($H - h$) in feet on the C scale opposite the focal length (f) in inches on the B scale and read the value of photo scale (S) in feet per inch on the C scale at the index.

To determine height above the ground $(H - h)$, multiply the focal length of the lens (f) by the photo scale (S).

> Put the photo scale value (S) in feet per inch on the C scale opposite the index line and read the altitude above the ground $(H - h)$ in feet on the C scale opposite the focal length of the lens (f) in inches on the B scale or opposite the focal length (f) in mm on the A scale.

To determine the focal length of the lens (f), divide the altitude above the ground $(H - h)$ by the photo scale (S).

> Put photo scale value (S) in feet per inch on the C scale opposite the index line and read the focal length of the lens (f) in inches on the B scale or in mm on the A scale opposite the altitude above the ground $(H - h)$ in feet on the C scale.

To determine differences in elevation as represented by the parallax equation

$$\frac{dh}{dx} = \frac{(H - h)^2}{fpS} *$$

divide altitude of the aircraft above the ground $(H - h)$ squared, by the focal length of the lens (f), times the distance between photo centers (p), times the photo scale (S).

> Put the value of the air base (p) in mm on the A scale or in inches on the B scale opposite the altitude of the aircraft above the ground $(H - h)$ on the C scale and read the difference in elevation per unit of measurement $\frac{dh}{dx}$ opposite the index in feet per 0.001 inch on the D scale, in feet per mm on the E scale or in feet per 0.01 mm on the F scale.

To determine the total elevational change (dh), multiply the units of parallax (dx) measured by feet per unit of parallax $\frac{dh}{dx}$.

> Put the value of the difference in elevation per unit of measurement $\frac{dh}{dx}$ on either the D, E or F scale, depending on unit of measurement, opposite the index. Set the hairline of the cursor over the number of units of parallax (dx) measured (read from metric scale on the HF-2) on the B scale and read the total elevation on either the D, E or F scale, depending on which scale was used for $\frac{dh}{dx}$.

* Derivation of the parallax equation is shown in the appendix and further examples of its application and definition of terms are shown in Section IV.

SECTION IV

Application Instructions

4-1. PREPARING AERIAL PHOTOGRAPHS. To prepare aerial photographs for stereo-viewing, accurately locate, and prick with a pin point, the principal point of each photograph made by the intersection of the lines connecting opposite fiducial marks. (See Figure 7)

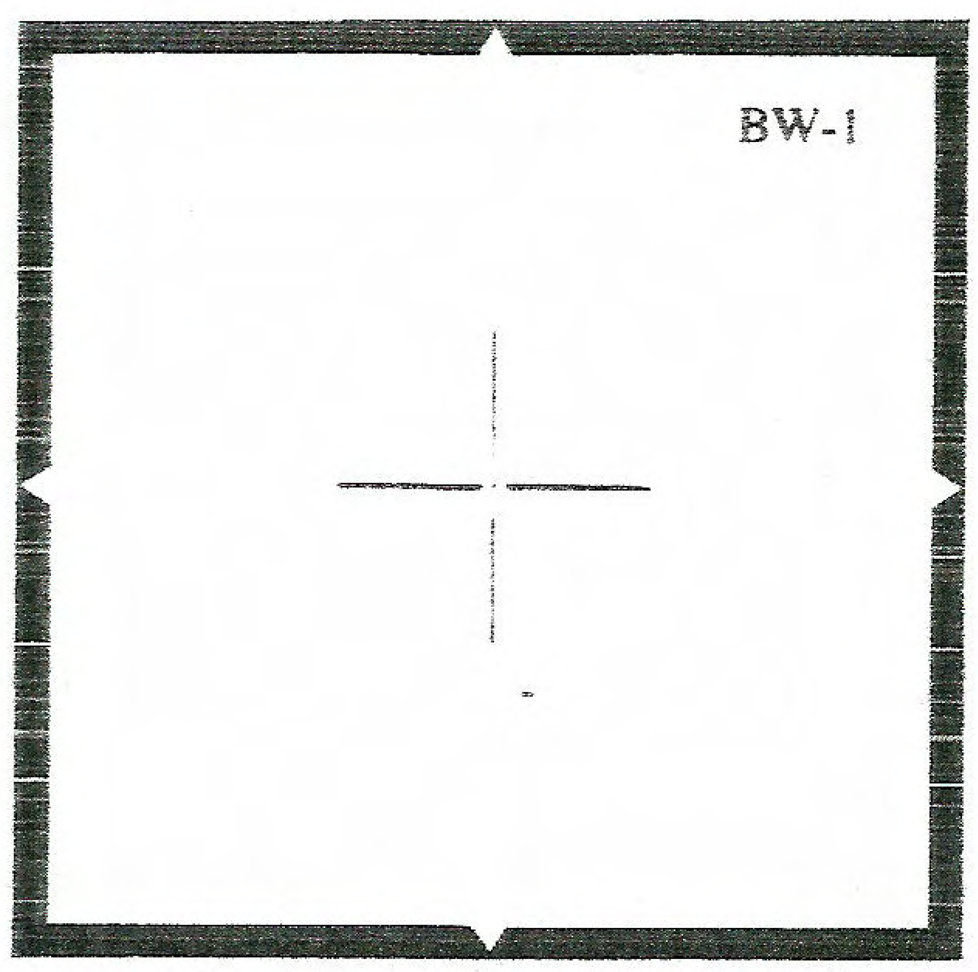


Figure 7 – Locating Principal Point

Transfer and prick the principal point of each photograph as it appears in the overlapping photo. The use of the stereoscope will enable the operator to identify these points more accurately.

Connect these two points with a very fine line on each photograph, beginning with the principal point, and extending through the transferred principal point to the edge of each photograph. This line is called the "flight line," and the distance between the two points is known as the "air base." The flight line can best be drawn by using a fine ball-point pen. (See Figure 8)

NOTE: The accuracy of the elevations obtained by the HF-2 depends in a large measure upon the accuracy with which the flight line and air base have been established.

Identify, prick, circle, and mark all known elevation points on both photos, preferably in red pencil.

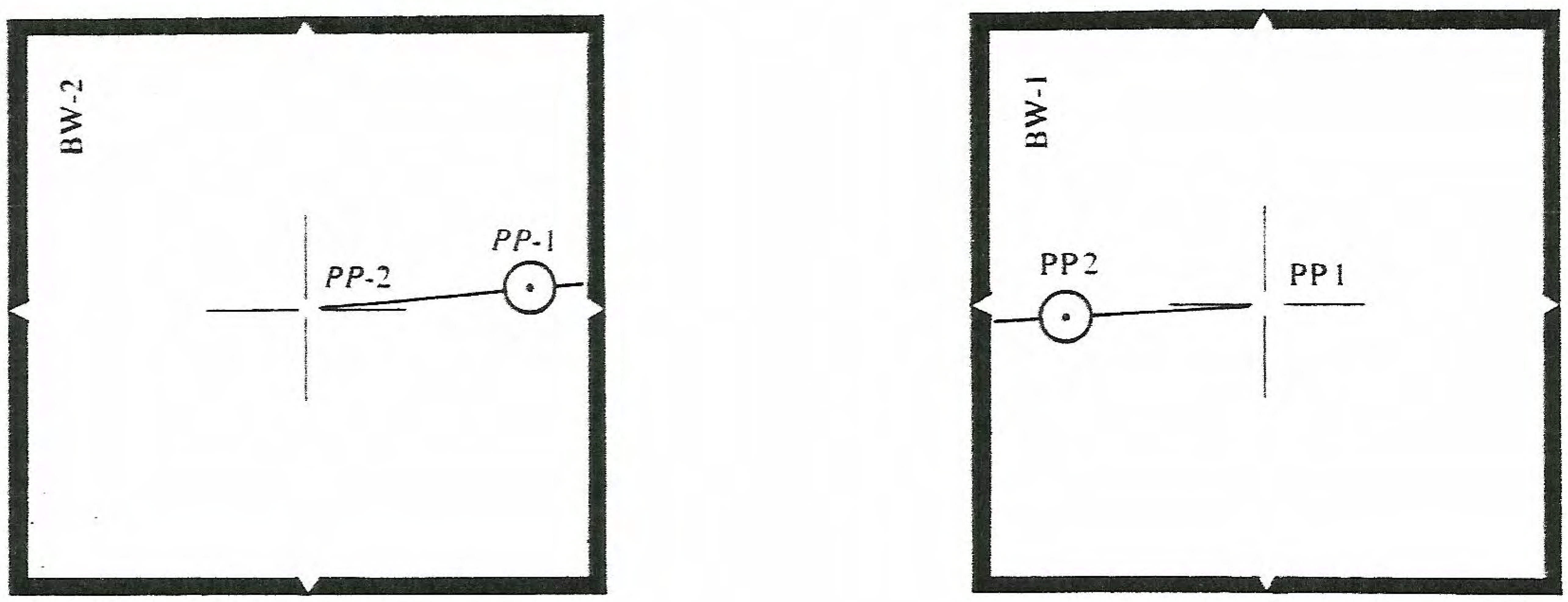


Figure 8—Transfer of Principal Points

4-2. PHOTOGRAPH ARRANGEMENT. The problem of adjusting the photos to the instrument is reduced to the following steps:

A. Setting up the left photo.
   (1) Align the two lenses, making sure both parallax dots are parallel to the base of the stereoscope. The knob should be set to read approximately 5 mm on the metric scale.
   (2) Tape the left photo securely along the left edge.

B. Setting up the right photo.
   (1) Place the right photo over the left, oriented in such a way that the "flight line" of both photos forms a continuous straight line.
   (2) Place the reference dot over the principal point of the left photograph.
   (3) Slide the right photo along the alignment of the "flight line" until the transferred principal point of the left photo, as it appears on the right photo, is under the moveable dot.
   (4) Re-check for the alignment of the "flight line" and correct if necessary.
   (5) Tape the right photo securely along the right edge.
   (6) Flip the photographs so that the left photo is on top of the right photo, and see that the flight lines coincide.

4-3. COMPUTING THE PARALLAX FACTOR. In order to determine the elevation of different objects, we must compute the parallax factor for each stereoscopic pair of photographs. This factor is affected by several things: (1) by flight altitude (H); (2) by height of datum plane (h) if not zero; (3) by air base measurements (p); and (4) by the focal length of the lens (f).

A. Obtain the relative altitude ($H - h$) at which the photos were taken by using the following formula:

$$\frac{1}{f} = \frac{S}{H-h} \quad \text{or} \quad H - h = \frac{(S)(f)}{1}$$

where f = focal length of camera in inches
S = scale of photo in feet per inch
$H - h$ = altitude of the aircraft above the ground in feet

further expressed as

$$\frac{1}{f\,\text{(inches)}} = \frac{S\ \text{(feet per inch)}}{H - h\ \text{(feet)}}$$

so that using f in inches and S in feet-per-inch the value of

$$H - h = \frac{(S)(f)}{1}$$

will be expressed in feet.

Example (1):

1:4800 or $\frac{4800}{12}$ is equal to 400 feet per inch = S

f = 6 inches $H - h = (400)(6)$ or $H - h$ = 2400 feet

Example (2):

1:12000 or $\frac{12000}{12}$ is equal to 1000 feet per inch = S

f = 8.25 inches $H - h = (1000)(8.25)$ or $H - h$ = 8250 feet

B. Calculate the actual ground distance (P) of the air base (p) (or ground distance between photo centers).

$$S = \frac{P}{p} \quad \text{or} \quad P = (p)(S)$$

where S = scale in feet per inch
p = air base in inches (distance between PP-1 and PP-2)
P = actual ground distance of the air base in feet

Example: S = 400 feet per inch
p = 2.5 inches
P = (2.5 inch)(400 feet per inch) = 1000 feet

C. Calculate the parallax factor by means of the following equation. (See appendix for derivation of parallax equation.)

$$\frac{dh}{dx} = \frac{(H-h)^2}{fP} \quad \text{or} \quad dh = \frac{(H-h)^2}{fP}\,dx$$

where dh = difference in elevation between a known elevation point and an unknown elevation point

dx = difference in parallax measured between the known and the unknown point

H = altitude (in feet) of the aircraft above mean sea level (MSL) or above the selected datum

h = height of known elevation (in feet) from sea level (MSL) or selected datum

f = focal length of the lens

P = ground distance (in feet) of the air base

Example: H = 2600 ft. above selected datum

h = 200 ft. above selected datum

f = 6 inches = 152.4 mm

P = 1000 ft.

$$dh = \frac{(2600 \text{ ft.} - 200 \text{ ft.})^2}{(152.4 \text{ mm})(1000 \text{ ft.})}\,dx$$

$$dh = \frac{5,760,000 \text{ ft.}^2}{152,400 \text{ mm-ft.}}\,dx$$

$$dh = 37.79 \text{ ft. per mm } (dx)$$

assuming dx = 1 mm

then dh = 37.79 ft.

37.79 is the parallax factor, equivalent to feet of elevation per millimeter of parallax as read on the metric scale of the HF-2.

D. The above equation can be further reduced if the datum is selected so that h equals zero, as follows:

$$\frac{dh}{dx} = \frac{.03937\,H}{p} \quad \text{or} \quad dh = \frac{.03937\,H}{p}\,dx$$

where .03937 = inches per millimeter

Example: H = 2400 ft.

p = 2.5 inches

$$dh = \frac{(.03937 \text{ in. per mm})(2400 \text{ ft.})}{2.5 \text{ inches}}\,dx$$

$$dh = \frac{94.488 \text{ inch-feet per mm}}{2.5 \text{ inches}} dx$$

assuming $dx = 1$ mm

then $dh = 37.79$ ft.

4-4. USE OF THE HEIGHT FINDER. When moving the HF-2 over the previously adjusted photographs, great care should be taken in maintaining the base of the stereoscope parallel to the "flight line."

When the spatial model is viewed through the instrument, the two parallax dots will be seen. If the eyes view the dots as one single dot, the dot may appear to be floating in the air. By turning the knob in a minus direction the "floating" dot will seem to drop onto the ground surface and then separate. In order to measure the elevation, it is *necessary* to have the two dots merge into one dot, so that it appears to be touching the ground surface. Care must be taken to keep the eyes focused on the ground features and not on the dots.

In learning how to bring the dots together, a new operator may use the following steps:

A. Close the right eye and observe where the left dot is in relation to the photo detail at a known elevation point.

B. Hold the instrument firmly and, by closing the left eye, make the right dot intersect the same detail as the left dot, by turning the knob in one direction or another.

C. When both dots intersect the same detail, the dot is near the ground surface and will appear merged into one when viewed stereoscopically.

D. In order to obtain a better accuracy in determining the elevation of a point, the floating dot should be raised and lowered several times to the ground around the point in question, and the readings of the metric scale recorded for each collimation. The average of the readings will determine the correct reading for this particular point.

4-5. FINDING THE ELEVATION OF AN UNKNOWN POINT. Difference in elevation between two points is determined by the difference in parallax readings (dx) obtained by the separate collimation of the point in question.

If the elevation of one of the points is known, it will be possible to find the elevation of the unknown point by the following steps:

A. Lower the dot several times to the surface of the known point; read and record the readings of the metric scale for each collimation. Take the average and record this value as x reading for the known point.

Example: $x = 2.73$

B. Repeat the same operation for the unknown point and record this value as x_1.

Example: $x_1 = 3.86$

C. Find the algebraic difference between the x reading of the unknown point and the known point. The resulting value with the proper sign is the difference in x parallax (dx) between the two points.

Thus $$dx = x_1 - x$$

Example: $x_1 = 3.86 \qquad x = 2.73$
$dx = 3.86 - 2.73 = +1.13$ (note plus)

Example: $x_1 = 2.08 \qquad x = 2.73$
$dx = 2.08 - 2.73 = -.65$ (note minus)

D. To find the difference in elevation (dh) multiply the "dx" value, with the proper sign, by the parallax factor as previously found by the formula

$$\frac{(H-h)^2}{fP}.$$

The sign of the difference in elevation is related to the elevation of the known point.

Example: $$\frac{(H-h)^2}{fP} = 37.79$$

as derived by the formula

$$dh = \frac{(H-h)^2}{fP}\,dx$$

in a previous example.

$dx = +1.13$

thus $dh = (37.79)(1.13) = 42.70$ feet

therefore $+42.70$ feet represents the difference in elevation between the two points.

Being the result of a positive sign the unknown point is higher than the known point.

E. To find the elevation value of the unknown point add, algebraically, the difference in elevation (dh) to the elevation of the known point.

Example: Elevation of known point = 643.5 feet
$dh = +42.7$

therefore elevation of the unknown point equals $643.5 + 42.7 = 686.2$ feet

If more than one elevation is known in the same pair of photographs, refer to the closest one to determine the elevation of the required point. A simple interpolation can also be made for points located between known elevations. Once the stereo pair of photographs (model) is properly set-up, the reading of the required points, if more than one, may be expedited by the use of a computation sheet where all the data can be properly entered in an orderly manner. A sample data sheet is shown in Figure 9.

These laborious hand computations can be eliminated by the use of the Abrams Computer which has incorporated the parallax factor involved in these computations.

Readings within 1 inch radius from a known elevation should produce an accuracy of one part in 300 of the flight height ($H - h$) on unrectified photographs. Where rectified photographs (corrected for tip, tilt and crab) are used, the accuracy can be greatly improved.

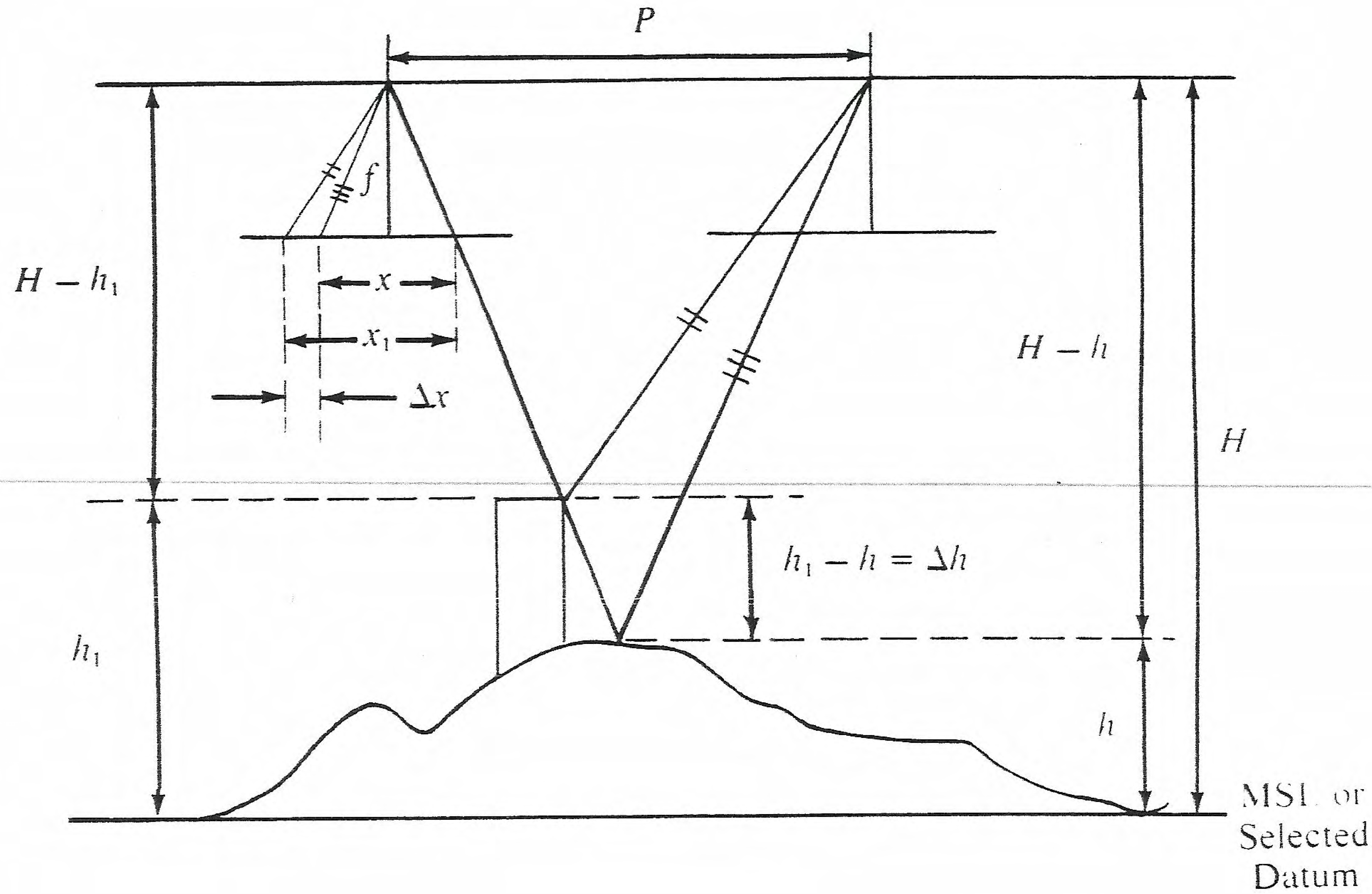
Project	N. N.	Photo Model Nos.	1 & 2
Photo Scale (S)	400 ft. per in.	Focal Length (f)	152.4 mm
Altitude of Aircraft (H)	2600 ft.	Air Base (p)	2.5 inches
Average Datum (h)	200 ft.		

$$dh = \frac{(H-h)^2}{fpS} = 37.79 \text{ ft. per mm } (dx)$$

Known Elev. Point	Elev. in Feet	Parallax Readings	Unknown Elev. Points	Parallax Reading x_1	dx	dh	Known Elev. + dh (200 + dh)
			a	3.62	+1.22	+46.1	246.1
			b	2.71	+ .31	+11.7	211.7
			c	4.24	+1.84	+69.5	269.5
Point "1"	200	2.40	d	3.18	+0.78	+29.5	229.5
			e	2.20	−0.20	− 7.5	192.5
			f	2.06	−0.34	−12.8	187.2
			g	3.54	+1.14	+43.1	243.1

Figure 9 – Data Sheet

APPENDIX
Derivation of Parallax Equation



Where P = distance between sequential camera positions (which equals actual ground distance of the air base)
f = focal length of lens
H = altitude of aircraft above datum
h = height of ground above datum
h_1 = height of object above datum
Δh = height of object
x = base height
x_1 = base height
Δx = base height differential

$$\frac{x}{f} = \frac{P}{H-h} \qquad \text{also} \qquad \frac{x_1}{f} = \frac{P}{H-h_1}$$

$$x = \frac{Pf}{H-h} \qquad \text{and} \qquad x_1 = \frac{Pf}{H-h_1}$$

$$x_1 - x = \frac{Pf}{H-h_1} - \frac{Pf}{H-h} \qquad \text{but} \qquad x_1 - x = \Delta x$$

$$\Delta x = \frac{Pf(H-h)}{(H-h_1)(H-h)} - \frac{Pf(H-h_1)}{(H-h_1)(H-h)} = \frac{Pf(H-h) - Pf(H-h_1)}{(H-h)(H-h_1)}$$

$$\Delta x = \frac{Pf[(H-h)-(H-h_1)]}{(H-h)(H-h_1)} = \frac{Pf(H-h-H+h_1)}{(H-h)(H-h_1)}$$

$$\Delta x = \frac{Pf(h_1-h)}{(H-h)(H-h_1)} \qquad \text{but} \qquad h_1 - h = \Delta h$$

$$\text{therefore} \qquad \Delta x = \frac{Pf\Delta h}{(H-h)(H-h_1)}.$$

Because the constant H is of predominant value with respect to h and h_1, being almost equal, the value of $(H-h)(H-h_1)$ is practically equal to $(H-h)^2$. Substituting for the equation $(H-h)^2$

$$\Delta x = \frac{Pf}{(H-h)^2}\,\Delta h$$

and consequently

$$\Delta h = \frac{(H-h)^2}{Pf}\,\Delta x.$$